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Novel approaches to the study of particle dark matter in astrophysics

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Novel Approaches to the Study of Particle Dark Matter in Astrophysics

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Abstract. A deep understanding of the role of the dark matter in the different astrophysical scenarios of the local Universe such as galaxies, represent a crucial step to describe in a more consistent way the role of dark matter in cosmology. This kind of studies requires the interconnection between particle physics within and beyond the Standard Model, and fundamental physics such as thermodynamics and statistics, within a fully relativistic treatment of Gravity. After giving a comprehensive summary of the different types of dark matter and their role in astrophysics, we discuss the recent efforts in describing the distribution of dark matter in the center and halo of galaxies from first principles such as gravitational interactions, quantum statistics and particle physics; and its implications with the observations.

INTRODUCTION

Particle dark matter candidates and its nature: a review

The first attempts to try to account for the needed dark matter (DM) in galaxies as suggested by observations (see [1] for a review), were by means of ordinary matter, i.e. the one involving protons, neutrons, electrons and photons, and usually named as *baryonic* DM. Already since the '70s, low massive stars ($0.01M_{\odot} < M < 0.08M_{\odot}$) as brown dwarfs¹ were proposed as a possible form of DM. However, at the beginning of the 21st century, with the technological revolution on Infrared astronomy conducted mainly by the 2MASS IR survey (see e.g. [2] for a review) or the HST IR NICMOS camera, it was measured an amount of brown dwarfs only as large as twice the amount of normal stars (i.e., stars with $M > 0.08M_{\odot}$); implying a negligible contribution to the DM budget. Low massive stars, as brown dwarfs or white dwarfs, enter in a wider category of DM candidates named MAssive Compact Halo Objects (MACHOs), which also includes primordial black holes (BHs). After several years of observations, in [3], it was concluded by means of statistical estimates that the mean mass of the MACHOs is between $0.15M_{\odot} < M < 0.9M_{\odot}$, showing that this objects can only correspond to about 20% of the DM in a typical halo, at 95% confidence level.

An independent and very strong limit to the total amount of baryon-composite DM, comes from primordial nucleosynthesis. The standard Big Bang model accounting for the first minutes of the life of the Universe, predicts with high accuracy, the abundance of light elements such as ^4He , ^3He , D, etc; being these elements a sensitive tracer of the mean baryon density of the Universe. Actually, in [4] the amount of these nuclides as obtained from recent observational data in different astrophysical environments (i.e., stars, galaxies, etc), is compared with its theoretically expected amount from BBN. Moreover, in [4] it is clearly shown the remarkably good agreement among primordial

¹The brown dwarfs are massive enough stars which never acquire high enough central temperature to start the hydrogen burning

nucleosynthesis theory, observational data of light nuclides, and the temperature fluctuation power spectrum of the CMB; all of them indicating a baryonic mass density contribution ρ_B to the total matter density of the Universe today ρ_0 , roughly of only a 15%, corresponding with a matter-energy fraction of $\Omega_B = \rho_B/\rho_0 \approx 0.045$.

Non-baryonic DM: HDM and CDM

Already since the beginning of the '80s (see e.g. [5] and references therein), after the original proposal by B. Pontecorvo in 1957 for the possible existence of neutrino oscillations, massive neutrinos were proposed as an appealing candidate to the DM problem.

HDM

In the first fractions of a second in the life of the Universe, at typical temperatures of about $kT \sim 1$ MeV, the time-scale of the weak-interactions t_{weak} , which prevented neutrinos escaping from the plasma of ν, e^\pm, γ , equals the time scale of the expansion of the Universe t_H (see, e.g., [6] for a detailed derivation). This condition, i.e. $t_H \sim t_{weak}$, implies the condition for decoupling of the neutrinos from the relativistic plasma. When the time-scale of the interaction becomes greater than the expansion rate (i.e. $t_{weak} > t_H$), the neutrinos are 'freeze out' and therefore become fundamental relics which will play an essential role in the structure formation of the Universe. At this early epoch, at the aforementioned decoupling temperature $kT \sim 1$ MeV, it can be shown that the ratio between the particle number density of neutrinos and anti-neutrinos to the photon number density is:² $n_{\nu\bar{\nu}}/n_\gamma = 6/11$. Thus, after the neutrino decoupling, and under the assumption that this ratio holds until now (i.e. at redshift $z \sim 0$), the photon number density can be directly calculated from the temperature of the CMB, $T \approx 2.7$ K, to obtain the actual neutrino mass-energy density, $\rho_{0(\nu\bar{\nu})} = m_\nu n_{0(\nu\bar{\nu})}$. Now, if this neutrino density is demanded to account for the total mass-energy density of the Universe today, $\rho_0 = 3H_0/(4\pi G)$ (with $H_0 \approx 70$ km/sMpc⁻¹ the Hubble constant today), i.e. $\rho_{\nu\bar{\nu}} \lesssim \rho_0$, it is found the following upper bound for the neutrino (and anti-neutrino) mass

$$m_\nu \lesssim 15 \text{ eV} \quad (\text{Gershtein\&Zeldovich} - \text{like}). \quad (1)$$

Similar upper bounds based on basically the same hypothesis used here, were originally found in [7, 8].

Cosmological DM in the form of ordinary neutrinos with masses up to few eV is known as HDM. However, if the rest mass of the neutrinos as estimated by laboratory experiments is ~ 0.1 eV (or even lower), then they cannot answer for all the DM present of the Universe (see [9] for a review). Indeed, the last observational results presented in the last Planck release combined with baryon acoustic data, gives $\Sigma_i m_{\nu_i} \lesssim 0.23$ eV, with a mass-energy density fraction from standard model neutrinos of $\Omega_\nu \lesssim 0.005$ (see e.g. [10]).

CDM

In 1977 B. W. Lee and S. Weinberg [11] realized that, if an hypothetical neutrino decoupled from the primordial plasma of ν, e^\pm, γ in a non-relativistic manner, oppositely as ordinary neutrino did, then, a completely different bound for the particle mass can be obtained. For this, they introduced an hypothetical massive stable neutral lepton $L^0 \equiv L$, such that $m_L \gg \text{MeV}$. Because of its large mass, such a neutrino species must decouple in a non-relativistic regime implying that the correspondent decoupling temperature from the primordial plasma can no longer be ~ 1 MeV, but higher. Otherwise, being so heavy, this hypothetical neutrinos would have much less number density at typical MeV decoupling temperatures (i.e. would reach a particle abundance $n_L \propto e^{-m/kT} \ll 1$). Thus, in order to treat the decoupling properly, they studied the evolution of particle's phase space distribution function with the Boltzmann equation in a Friedam-Robertson-Walker (FRW) model. This is, they solved the equation $\hat{L}[f] = C[f]$ (\hat{L} the covariant Liouville operator and C the collision operator), for $n_L \propto \int d^3p f(E, t)$, with $f(E, t)$ following the Maxwell-Boltzmann statistics. To deal with the collision term, they treated the annihilations cross section in the 'V-A' model of charged weak interactions (i.e. $\langle \sigma v \rangle = N_A G_F^2 m_L^2 / (2\pi)$), taking the fudge factor $N_A = 14$ subject to all possible annihilation channels of $L^0 \bar{L}^0$ for such a massive particle. While, to deal with the energy density of the Universe in the high temperature primordial plasma³, they took $N_F = 4.5$ which accounts for the different particle species that

²For this calculation it has been assumed an ultra-relativistic approximation for the neutrinos (all types), zero chemical potential in the distribution functions of the bosons (γ particles) and fermions, and a complete annihilation of the e^\pm plasma into photons which re-hit the pre-existing photon radiation. This last assumption implies the following relation between the photon and neutrino-antineutrino temperatures $T_\gamma = T_{\nu\bar{\nu}}(11/4)^{1/3}$.

³With $\rho = N_F a T^4 \propto H(T)^2$, being ρ dominated by the highly relativistic particles ν, γ, e^+ and e^- .

contributes effectively to ρ . Once with all these considerations they solve numerically the Boltzmann equation for the time dependent particle abundance $n_L(t)$, which after fulfilling the ‘freeze out’ condition (i.e. the interaction rate \sim expansion rate), they obtain for the present heavy neutrino and antineutrino density $\rho_{0(L)} \propto m_L^{-1.85}$. They finally compared this neutrino density to the actual matter density in the Universe (i.e. $\rho_{0(L)} \lesssim \rho_0$), to obtain

$$m_\nu > 2 \text{ GeV}/c^2 \quad (\text{Lee\&Weinberg}). \quad (2)$$

Where it can be shown that the decoupling temperature in correspondence to this bound is $\gtrsim 140$ MeV, this is, well before the relativistic decoupling of the ordinary neutrinos. It is important to note that the Lee and Weinberg bound (2) is a lower limit, while the Gershtein and Zeldovich-like bound is an upper limit. Indeed, by solving the Boltzmann equation in a FRW model for stable and weakly interacting massive neutrinos which decouples in thermal equilibrium, either in a non-relativistic regime or a relativistic one, it can be more generally shown that when the energy density of the Universe is neutrino dominated, the only two possible mass bounds are given by equations (1) and (2) (see e.g. [12] Fig. 5.2). This is, only HDM and CDM paradigms are possible within a weakly interacting massive neutrino thermal decoupling context.⁴

The success of the standard Λ CDM model relies in the consistency when contrasted with independent observational data sources as: i) The observed distribution of galaxies in the nearby (~ 150 Mpc) Universe as obtained in [14]; ii) The baryon acoustic peaks in the power spectrum of galaxies as obtained by the teams of the 2dF Galaxy Redshift Survey and the Sloan Digital Sky Survey (SDSS) [15, 16]; iii) The temperature fluctuations in the full CMB radiation spectrum as obtained by the Planck satellite [17]. Nevertheless, some important problems remains at small scale structures ($\sim 10^2 - 10^3$ pc). Among the most relevant discrepancies between the effects of CDM in structure formation at these small scales, and observations, we quote: i) a predicted overabundance of satellite galaxies; ii) a prediction of cuspy halos which gives rise to the often called core-cusp discrepancy, and iii) a prediction of *too high* dispersion velocities in dwarf satellite galaxies. All these related effects can be understood mainly in terms of the relatively *too small* free streaming length of the cold particles, which where non-relativistic since their decoupling and therefore implying *too many* and *too clumped* gravitationally bounded structures when compared with observations. See for example [13, 18, 19, 20], for discussion on all these problems.

WDM: a promising alternative

Since the '80s, and contemporaneously to the two proposed forms of DM discussed before, another proposal for the dark candidates with masses in the keV region appeared as an intriguing and appealing candidate, but this time from astrophysical constraints. As we are dealing here with collisionless massive particles, constraints on its phase space density can be set by knowing its precise evolution from the time of decoupling up to the approximate time of virialization of a DM halo. The starting point to this discussion is the famous work of S. Tremaine and J. Gunn in 1979 [21]. They used the Liouville's theorem plus the concept of violent relaxation introduced in [22] to argue that, 1) the primordial coarse-grained phase-space density⁵ must never increase, but only decrease, implying therefore a maximum value for the late-time coarse-grained phase space density ending in a DM halo. Just after decoupling, the microscopic fine-grained phase space DF $f(p)$ coincides with the coarse-grained DF due to the absence of phase mixing effects. Thus, they considered massive neutrinos which decoupled in a relativistic regime with an initial fine-grained phase space density, denoted here as Q_{fg} , which at its maximum is given by⁶ $Q_{max} = 4(g_\nu h^{-3}/2)$, for the whole two neutrinos species with the corresponding anti-neutrinos (i.e. $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$). They further proposed the simplest late-time velocity distribution for the formed DM halos: the Maxwellian distribution, leading to an isothermal-sphere-like solution for the hydrostatic equilibrium condition. More specifically, they further assumed: 2) that the central region of these bound systems of self-gravitating massive neutrinos are described by classical Newtonian solutions resembling isothermal gas spheres; 3) that their velocity distribution is Maxwellian and the maximum phase-space density of that

⁴It is important to mention that within this framework higher upper mass bounds (i.e. in the keV regime), can be found for hypothetical fermions decoupling relativistically and in thermodynamic equilibrium, by setting the energy scale of decoupling at electro-weak scales, i.e. at $kT_d \sim 100$ GeV (see e.g. [13], equation 2.49).

⁵A coarse-grained DF is defined as the average of the (microscopic) fine-grained DF $f(p)$ over a finite volume, containing a measurable large enough number of particles.

⁶The primordial fine-grained occupation number for neutrinos is given by the Fermi-Dirac distribution function $f(p) = [\exp(e(p) - \mu)/kT + 1]^{-1}$, and its fine-grained phase-space density by $Q_{fg} = (g_\nu/h^3)f(p)$; with $e^2 = (pc)^2 + m^2c^4$ and g_ν the spin degeneracy. The Fermi-Dirac occupation number acquires its maximum at $p = 0$ (i.e., $f_{max} = [\exp(mc^2 - \mu)/kT + 1]^{-1}$). In the case of $\mu = 0$ and for ordinary light neutrinos decoupling at $T_d \sim 1$ MeV (i.e. $mc^2/kT_d \rightarrow 0$) as considered in [21], the resulting maximum occupation number is $1/2$. Thus, the maximum phase-space density is $Q_{fg,max} \equiv Q_{max} = 4(g_\nu h^{-3}/2)$, where the factor 4 overall corresponds to the four kinds of neutrino considered.

self-gravitating configuration is given by $Q_{max}^{iso} = \rho_0 m^{-4} (2\pi\sigma^2)^{-3/2}$, m being the neutrino mass, ρ_0 the central density, σ the one dimensional velocity dispersion and the King radius being $r_K^2 = 9\sigma^2/4\pi G\rho_0$. Then, relying in the fact that the primordial phase-space can only decrease as stated in 1), they made the comparison $Q_{max}^{iso} < Q_{max}$, to get the lower limit in the particle mass:

$$m > 101 \left(\frac{1}{g_v} \frac{100 \text{ km/s}}{\sigma} \frac{1 \text{ kpc}}{r_K} \right)^{1/4} \text{ eV}/c^2. \quad (3)$$

In the case of typical spiral galaxies with $\sigma \sim 100 \text{ km/s}$ and $r_K \sim 1 \text{ kpc}$, the expression (3) sets a keV-ish lower bound for the massive neutrino (with i.e. $g_v = 2$), excluding in an elegant way (from phase-space constraints applied to DM halos) the eV mass-scales as obtained in the above HDM section. More important is the fact that, if this new keV mass scale particles are pretended to provide the main DM budget in the Universe, the particles has to decouple either out of thermodynamic equilibrium or at electro-weak scales. Otherwise only the eV-ish or GeV-ish mass scales are allowed as already explained.

The central characteristic of keV neutrinos regarding structure formation at small scales, is that having higher free streaming lengths than CDM particles at the moment of structure formation, then, less central DM halo concentrations are produced, solving the discrepancies arising within Λ CDM (see e.g. [23] for a numerical simulation approach, and [27] for a first principle physics approach). Besides the Tremaine & Gunn limit (3), a more detailed comparison between high resolution data in galaxies and theory involving phase-space DFs, can be performed. Indeed, a comparison between observed galaxy rotation curves and its matter density profiles against the ones arising from an underlying *equilibrium* phase space DF $f(r, p)$, has been extensively studied in the literature in the last few decades. Nevertheless, this approach faces a central problem: actually, it is an unknown thing how to determine the functional form $f(r, p)$ for a late-time microscopic phase-space density of DM in a virialized halo, due to the complex relaxation processes involved. We mention in next two different important attempts to attack this issue: **I**) To propose a fundamental physical principle from which to obtain $f(r, p)$; **II**) To propose a phenomenological expression for $f(r, p)$ (either first principle physics supported or not) from which the corresponding density profiles fit as best as possible the observations, and further compare it with the fitted profiles arising from numerical N-body simulations. While the main results within the point **I**) can be found in the pioneering work of Lynden-Bell [22], the present work is based on the approach **II**). Interestingly, the central hypothesis regarding a possible form for $f(r, p)$ of Fermi-Dirac-type used within the second approach, appears to be the more natural one when considering the results found within the first approach, as clearly explained in [27] and references therein.

A NOVEL CORE-HALO DISTRIBUTION OF DARK MATTER IN GALAXIES

Within the realm of non-baryonic DM in the form of collisionless massive fermions, and following [24, 25, 26, 27, 28, 29] and references therein, we present here the model of self-gravitating and semi-degenerate fermions including relativistic effects and its different successful applications to astrophysics. By solving in the more general way the Tolman Oppenheimer Volkoff (TOV) equations for hydrostatic equilibrium of a thermal and semi-degenerate fermion gas with given boundary conditions in agreement with galactic halo observables, we show a novel possible distribution of dark matter in galaxies dealing with distance-scales from mpc up to Mpc, being able to segregate different marked physical regimes: 1) an inner core of almost constant density governed by degenerate quantum statistics; 2) an intermediate region with a sharply decreasing density distribution followed by an extended plateau, implying quantum corrections; 3) a decreasing density distribution $\rho \propto r^{-2}$ leading to flat rotation curves fulfilling the classical Boltzmann statistics. We further show how sensitive is the particle mass when the solutions are asked to fulfill all the accessible observables on galaxy scales, and present different astrophysical applications such as the excellent agreement of the model with the observed constant Universal correlation between the dark matter surface density and their one-halo scale-lengths. It is presented in particular, a natural and novel way to predict the DM content in the very central part of dwarf spheroidal and spiral galaxies. More interestingly, it is analyzed up to which extents these central DM concentrations can be interpreted as an alternative to the central BH paradigm, and how, when equilibrium solutions allows for it, the keV fermionic DM particle appears as a natural candidate.

Explicitly, the density ρ and pressure P of the fermionic system are given by (with the particle helicity $g = 2$) $\rho = m \frac{2}{h^3} \int f(p) \left[1 + \frac{\epsilon(p)}{mc^2} \right] d^3 p$ and $P = \frac{1}{3} \frac{2}{h^3} \int f(p) \left[1 + \frac{\epsilon(p)}{mc^2} \right]^{-1} \left[1 + \frac{\epsilon(p)}{2mc^2} \right] \epsilon d^3 p$, where the integration is extended over all the momentum space and $f(p) = (\exp[(\epsilon(p) - \mu)/(kT)] + 1)^{-1}$. Here $\epsilon(p) = \sqrt{c^2 p^2 + m^2 c^4} - mc^2$ is the particle kinetic energy, μ the chemical potential with the particle rest-energy subtracted off; T is the temperature, k is the

Boltzmann constant, h is the Planck constant, c is the speed of light, m is the ‘ino’s particle mass. We do not include the presence of anti-fermions, i.e. consider temperatures that satisfy $T \ll mc^2/k$.

We write the system of Einstein equations in the spherically symmetric metric $g_{\mu\nu} = \text{diag}(e^\nu, -e^\lambda, -r^2, -r^2 \sin^2 \theta)$, where ν and λ depend only on the radial coordinate r , together with the thermodynamic equilibrium conditions of Tolman [30], and Klein [31] $e^{\nu/2}T = \text{const.}$ and $e^{\nu/2}(\mu + mc^2) = \text{const.}$ respectively, in the following dimensionless way,

$$\frac{d\hat{M}}{d\hat{r}} = 4\pi\hat{r}^2\hat{\rho} \quad \frac{d\theta}{d\hat{r}} = \frac{\beta_0(\theta - \theta_0) - 1}{\beta_0} \frac{\hat{M} + 4\pi\hat{r}^3}{\hat{r}^2(1 - 2\hat{M}/\hat{r})} \quad \frac{d\nu}{d\hat{r}} = \frac{\hat{M} + 4\pi\hat{r}^3}{\hat{r}^2(1 - 2\hat{M}/\hat{r})} \quad \beta_0 = \beta(r)e^{\frac{\nu(r) - \nu_0}{2}}. \quad (4)$$

There, the following dimensionless quantities were introduced: $\hat{r} = r/\chi$, $\hat{M} = GM/(c^2\chi)$, $\hat{\rho} = G\chi^2\rho/c^2$ and $\hat{P} = G\chi^2 P/c^4$, where $\chi = 2\pi^{3/2}(\hbar/mc)(m_p/m)$ is the dimensional factor which has unit of length and scales as m^{-2} ; with $m_p = \sqrt{\hbar c/G}$ the Planck mass, and the temperature and degeneracy parameters, $\beta = kT/(mc^2)$ and $\theta = \mu/(kT)$, respectively.

The system (4) is integrated for given initial conditions at the center, $r = 0$: $M(0) = 0$, $\nu(0) = 0$, $\theta(0) = \theta_0$ and $\beta(0) = \beta_0$ in order to be consistent with the observed dark matter halo mass $M(r = r_h) = M_h$ and radius r_h , defined in our model at the onset of the flat rotation curves. Equations (4) are solved for selected values of θ_0 and m , corresponding to different degenerate states of the gas at the center of the configuration. The value of β_0 is actually an eigenvalue which is found by a trial and error procedure until the observed values of v_h and M_h at r_h are obtained. In Figure 1 it is shown the density profiles and the rotation curves as a function of the distance for a wide range of parameters (θ_0, m) , in the case of boundary conditions typical of spiral galaxies (i.e. $r_h = 25$ kpc, $v_h = 168$ km/s, $M_h = 1.6 \times 10^{11} M_\odot$ [32]) are exactly fulfilled.

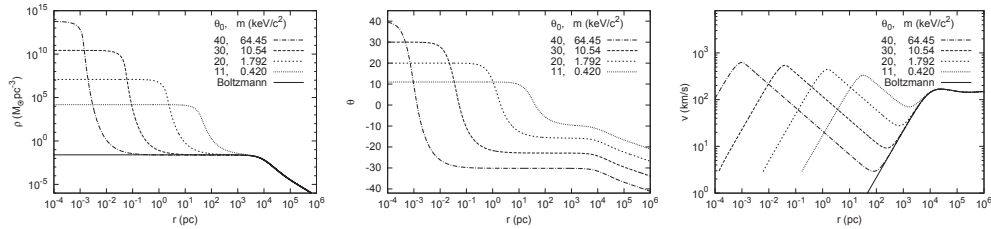


FIGURE 1. Mass density and degeneracy parameter profiles for specific ‘ino’ masses m and central degeneracies θ_0 fulfilling the observational halo constraints typical of spiral galaxies M_h and v_h at r_h . The density solutions are contrasted with a Boltzmannian isothermal sphere with the same halo properties. It is clear that all the configurations, for any value of θ_0 and corresponding m , converge for $r \gtrsim r_h$ to the classical Boltzmannian isothermal distribution.

A necessary condition for the validity of this quantum treatment of the core is that the interparticle mean-distance, l_c , be smaller or of the same order, of the thermal de Broglie wavelength of the inos, $\lambda_B = h/\sqrt{2\pi mkT}$. This result is clearly shown in [27], which is fulfilled in all the galactic types studied, from dwarf to big spirals. Defining the core mass, the circular velocity at r_c , and the core degeneracy as $M_c = M(r_c)$, $v_c = v(r_c)$ and $\theta_c = \theta(r_c)$, respectively; in Table 1 it is shown the core properties of the equilibrium configurations in spiral galaxies, for a wide range of (θ_0, m) . For any selected value of θ_0 we obtain the correspondent ‘ino’ mass m to fulfill the halo properties M_h and v_h at r_h , after the above eigenvalue problem of β_0 is solved.

θ_0	m (keV/c ²)	r_c (pc)	$M_c(M_\odot)$	v_c (km/s)	θ_c
11	0.420	3.3×10^1	8.5×10^8	3.3×10^2	2.1
25	4.323	2.5×10^{-1}	1.4×10^7	4.9×10^2	5.5
30	10.540	4.0×10^{-2}	2.7×10^6	5.4×10^2	6.7
40	64.450	1.0×10^{-3}	8.9×10^4	6.2×10^2	8.9

TABLE 1. Core properties for different equilibrium configurations fulfilling the halo parameters $v_h = 168$ km/s, $M_h = 1.6 \times 10^{11} M_\odot$ at $r_h = 25$ kpc, typical of spiral galaxies.

From the results presented in Table 1 and Figure 1 it is possible to conclude that a compact degenerate core mass $M_c \sim 4 \times 10^6 M_\odot$ is definitely possible corresponding to an ‘ino’ of $m \sim 10$ keV/c², and therefore an alternative inter-

pretation to the black hole in SgrA* appears as an interesting possibility. However, the core radius of the configuration is larger by a factor $\sim 10^2$ than the pericenter of the closest observed star to Sgr A*, i.e. the S2 star [33]. Therefore, this means that the semi-degenerate quantum core in the density profile of a MW like galaxy is not compact enough to answer for its orbit. Nevertheless, for an ino mass of $m \sim 10 \text{ keV}/c^2$ ($\theta_0 = 30$), the very low temperature of the dense quantum core is already a small fraction of the Fermi energy (i.e. $\lambda_B > l$), where additional interactions between the inos should arise, affecting the mass and radius of the new denser core depending on the interaction adopted ⁷. Indeed, we will show here how this novel idea first given in [29], can lead to higher compactness of the quantum core.

If one considers also different sets of physical dark matter halos typical of dwarf spheroidal galaxies: $r_h = 0.6 \text{ kpc}$; $v_h = 13 \text{ km/s}$; $M_h = 2 \times 10^7 M_\odot$, (e.g. from [45]); and $r_h = 75 \text{ kpc}$; $v_h = 345 \text{ km/s}$; $M_h = 2 \times 10^{12} M_\odot$ typical of big spiral galaxies [46], a remarkable outcome arises within the application of our fermionic model. This is, for the same ino mass of $m \sim 10 \text{ keV}/c^2$ in all cases, we obtain respectively core masses $M_c \sim 10^4 M_\odot$ and radii $r_c \sim 10^{-1} \text{ pc}$ for dwarf galaxies, and core masses $M_c \sim 10^7 M_\odot$ and radii $r_c \sim 10^{-2} \text{ pc}$ for big spirals. This suggest a possible alternative to intermediate ($\sim 10^4 M_\odot$) and more massive ($\sim 10^{6-7} M_\odot$) black holes, thought to be hosted at the center of the galaxies. Moreover, it allow us to obtain out of first principles, a possible universal relation between the dark matter halos and the super massive dark central objects which is comparable with an observed correlation found by Ferrarese in [36], but without assuming the BH hypothesis. For a fixed ino mass $m = 10 \text{ keV}/c^2$, we found the M_c - M_h correlation law (see [27] for more details)

$$\frac{M_c}{10^6 M_\odot} = 2.35 \left(\frac{M_h}{10^{11} M_\odot} \right)^{0.52}, \quad (5)$$

valid for core masses $\sim [10^4, 10^7] M_\odot$ (corresponding to dark matter halo masses $\sim [10^7, 10^{12}] M_\odot$).

We give in next a brief summary to the work [29], where for the first time, they were analyzed the possible consequences caused by a detailed *self-interacting field theoretic* (relativistic) model of Majorana fermions, with vector type interactions and fermion rest masses in the keV/c^2 range playing the role of WDM in galaxies. In particular, the collisionless nature of the dark matter fermions at halo scales were maintained, to study the two-particle self-interaction effects for different interaction strengths, but only in the (sub-pc) region, where the dense fermionic quantum core arises [27]. The relevance of self-interactions in ultra-cold fermionic-particle collisions has been already shown in laboratory [35], for example, for (effective) Fermi gases (e.g. ^6Li) at temperatures of fractions of the Fermi energy, thus motivating our central assumption. We start the minimal extension of the model of the lightest-right-handed-fermion sector of νMSM [37, 38, 39], which plays the role of dark matter, by introducing on the basis of a phenomenological effective picture, self-neutrino interactions through a massive vector meson V_μ mediator.

The Lagrangian of the right-handed neutrino sector, including gravity, reads (we use units $\hbar = c = 1$):

$$\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_{N_{R1}} + \mathcal{L}_V + \mathcal{L}_I \quad (6)$$

where $\mathcal{L}_{GR} = -\frac{R}{16\pi G}$, $\mathcal{L}_{N_{R1}} = i\bar{N}_{R1}\gamma^\mu \nabla_\mu N_{R1} - \frac{1}{2}m\bar{N}_{R1}N_{R1}$, $\mathcal{L}_V = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu$, and $\mathcal{L}_I = -g_V V_\mu J_V^\mu = -g_V V_\mu \bar{N}_{R1}\gamma^\mu N_{R1}$. With R the Ricci scalar for the static spherically symmetric metric background $g_{\mu\nu} = \text{diag}(e^\nu, -e^\lambda, -r^2, -r^2 \sin^2 \theta)$, where e^ν and e^λ depend only on the radial coordinate, r . The quantity m is the mass of the sterile neutrino, $\nabla_\mu = \partial_\mu - \frac{i}{8}\omega_\mu^{ab}[\gamma_a, \gamma_b]$ is the gravitational covariant derivative acting on a Majorana spinor, with ω_μ^{ab} the spin connection and $[,]$ the commutator. The vector-meson mass is m_V , whose microscopic origin is not discussed here, and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, where the ‘‘Lorentz gauge condition’’ $\partial^\mu V_\mu = 0$ has been applied for the vector meson (VM) field V_μ ; while the (effective) fermion-fermion interaction is modeled through the minimal Yukawa coupling term, governed by the coupling constant g_V .

The generalized Tolman and Klein conditions with the inclusion of a VM extra field (see [40] for details), are $e^{\nu/2}T = \text{const.}$ and $e^{\nu/2}\mu + g_V V_0 = e^{\nu/2}(\mu + C_V n) = \text{const.}$, with the parameter $C_V \equiv g_V^2/m_V^2$ encoding the information of the strength of the coupling of the effective interactions of the fermions, and the mass of the vector meson mediator. The equations of motion obtained from (6) for the spherically symmetric metric, and using the generalized Tolman and Klein conditions with the inclusion of a VM extra field are:

$$\frac{d\hat{M}}{d\hat{r}} = 4\pi\hat{r}^2\mathcal{E} \quad \frac{d\nu}{d\hat{r}} = 2\frac{\hat{M} + 4\pi\hat{r}^3}{\hat{r}^2(1 - 2\hat{M}/\hat{r})} \quad \frac{d\theta}{d\hat{r}} = -\frac{1}{2\beta}\frac{d\nu}{d\hat{r}}\left(1 + \frac{C_V m^2}{4\pi^3}\hat{n} - \frac{C_V m^2}{4\pi^3}\beta\frac{d\hat{n}}{d\beta}\right) \quad \beta = \beta_0 e^{\frac{\nu_0 - \nu(r)}{2}}, \quad (7)$$

⁷This is analogous for instance to the case of neutron stars, where nuclear fermion interactions strongly influence the mass-radius relation (see, e.g., [34])

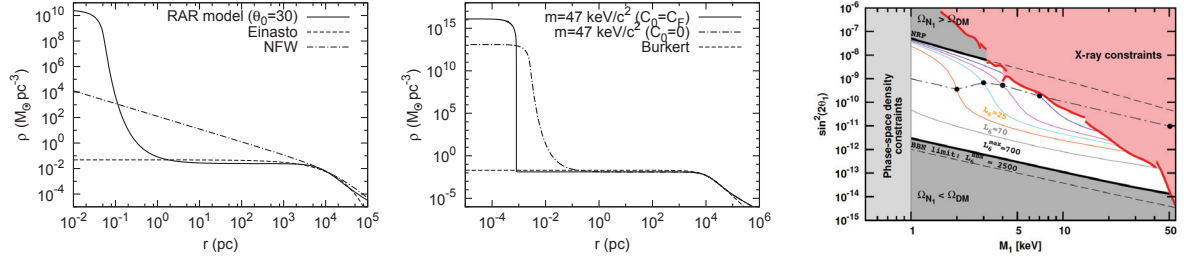


FIGURE 2. *Left:* A solution of the RAR model $C_V = 0$ contrasted with NFW and Einasto profiles usually adopted in the literature, showing the similar behaviour at halo scales. *center:* mass density profiles for $m = 47 \text{ keV}/c^2$ in a given interaction regime $C_V \equiv C_0 = C_F$ (being C_F the Fermi constant of the weak interaction for reference), where core and halo observational constraints M_c and r_c are fulfilled; compared with the non-interacting case ($C_V = 0$) for the same ino mass. We also show for comparison the two parametric Burkert profile $\rho_B/[(1 + r/h)(1 + (r/h)^2)]$ with $\rho_B = 2 \times 10^{-2} M_\odot/\text{pc}^3$ and $h = 10 \text{ kpc}$, which is the best dark matter halo fit of the Milky Way according to [41]. *Right:* lightest sterile neutrino mass range of the νMSM model, consistent with all the current astrophysical and cosmological constraints [39], showing the compatibility of the mass range $47 \leq m \leq 50 \text{ keV}/c^2$ obtained within our model.

where we have used the relativistic mean-field approximation (RMF) for the VM-field $V_\mu \equiv V_0 = \frac{g_V}{m_V} J_0^V$ ($J_\mu^V \equiv J_O^V$) with the notations $\langle V_0 \rangle \equiv V_0$ and $\langle J_0^V \rangle \equiv J_0^V = n u_0$, where $u_0 = e^{v/2}$ is the time-component of the (average) future-directed four velocity vector, and we have used the normalization condition $u^\mu u_\mu = 1$. It is important to notice that the system (7) coincides with the original version (4), in the case of no interactions ($C_V = 0$).

On astrophysical grounds and in order to apply the above extended model to our galaxy, the idea is to propose the dense DM core as an alternative to the usual central massive BH scenario, while at the same time, the outer distribution must agree with the observationally inferred DM halo constraints. For this we adopt a complete phenomenological analysis by studying the maximum possible range of interactions strengths $C_V \equiv C_0$ in the quantum core to be in agreement with the minimal required compactness consistent with the BH alternative. The required central compactness is set by the observations of the S2 star orbit, being the minimum required density of the DM core: $\rho_0 \sim 10^{16} M_\odot/\text{pc}^3$ (and up to $\sim 10^{23} M_\odot/\text{pc}^3$), achieved when $M_c = 4.4 \times 10^6 M_\odot$ inside $r_c \lesssim r_{c(S2)} = 6 \times 10^{-4} \text{ pc}$, being $r_{c(S2)}$ the S2 star pericenter. This approach is done such that the (classical) diluted halo remains unaffected by the interactions (i.e. $C_V = 0$ for $r > r_c$). The results are summarized in Figure 2. See [29], for the detailed procedure.

In summary, motivated by the RMF theories successfully applied to the self-interacting fermions in astrophysical compact objects, a model of self-interacting right-handed neutrinos (more specifically sterile-neutrinos within the νMSM) was developed by introducing a vector boson field V_μ , interacting (in the minimal coupling choice) with the fermionic field. Interestingly, for any interaction regime studied, a sterile-neutrino mass range between few 10^1 keV till few 10^2 keV is allowed by the core-halo observables. The results here presented, i.e. that an sterile neutrino with few tens of keV is selected *independently* from both galactic and DM astro-particle physics studies, points towards an important role of the right-handed neutrinos in the cosmic structure.

Universal mean surface density of dark matter halos

A central outcome of the fermionic RAR model presented in the former section is to be in excellent agreement with the observational and Universal empirical correlation between the surface density of DM halos and their one-halo scale-lengths as found in [42], and contrary to the case of bosons [43].

Specifically speaking, the so-called ‘central’ surface density of galaxy DM halos $\Sigma_{0D} = r_0 \rho_{0h}$, where r_0 and ρ_{0h} are the core radius of the halo (or halo-scale-length) and central (halo) density respectively, is roughly a constant independently of the galaxy luminosity [42] (see Figure 3, left panel). In that significant paper, a sample of several hundreds of rotation curves allowing for mass models in a very wide range of galaxy types from dwarf to elliptical galaxies, was analyzed under the assumption that each DM halo follows a Burkert profile $\rho_B(r)$, to obtain the following relation

$$\Sigma_{0D} \equiv r_0 \rho_{0h} = 140_{-30}^{+80} M_\odot \text{pc}^{-2}, \quad (8)$$

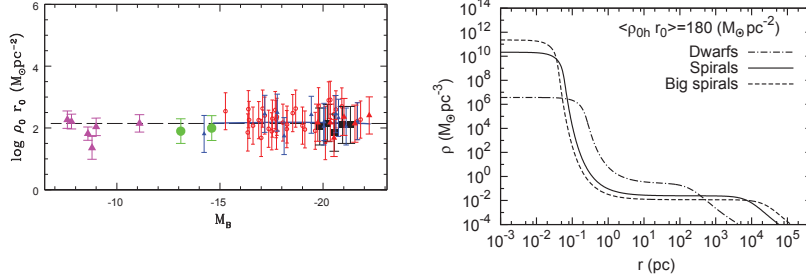


FIGURE 3. *Left:* The ‘central’ surface density of galaxy DM halos $\Sigma_{0D} = r_0 \rho_{0h}$ is approximately constant for many different galaxy types, independently of its luminosity as proven from observations in [42]. *right:* Mass density profiles for specific ‘ino’ masses m and central degeneracies θ_0 fulfilling the DM halo observational constraints for each galaxy type here considered, having an averaged DM surface density $\langle \rho_{0h} r_0 \rangle$ in agreement with observations.

where the one-halo scale length r_0 is now the Burkert radius and ρ_{0h} the central halo density. The empirical relation (8) clearly implies that the bigger the halo size of the galaxy, the lower its central halo density, following a more or less a precise recipe. Therefore, an evident question to be answered is if our model of self-gravitating keV fermions is able to follow the same kind of relation (8), e.g. for all the different galaxy types here considered. The answer is yes, and it is clear from the following didactic analysis (see [44] for more details).

We present in next a direct compatibility between the theoretical model of semi-degenerate fermions and the observed relation (8), by taking the galaxy parameters (halo mass and circular velocity at the halo radius) for the typical dwarfs, spirals and big spirals as considered here and in [27]. Thus, I take the following physical dark matter halos parameters: $r_h = 0.6$ kpc, $v_h = 13$ km/s, $M_h = 2 \times 10^7 M_\odot$ for typical dwarf spheroidal galaxies [45]; $r_h = 25$ kpc, $v_h = 168$ km/s, $M_h = 1.6 \times 10^{11} M_\odot$ for typical spiral galaxies [32]; and $r_h = 75$ kpc, $v_h = 345$ km/s, $M_h = 2 \times 10^{12} M_\odot$ for big spiral galaxies [46].

Then, for a mass of $m = 10$ keV/ c^2 , we first provide the density profiles such that all the different galaxy parameters given above (v_h , and M_h at r_h) are fulfilled in each representative case⁸, to then calculate from our model the DM surface density $\rho_{0h} r_0$ inferred from observations in [42] and shown in Figure 1. Finally it is also calculated and presented in the Figure the averaged surface density $\langle \rho_{0h} r_0 \rangle$ which clearly show the good agreement with the empirical Universal law (8).

Super massive dark compact objects at the center of galaxies

The simplest version of the fermionic DM model presented above, can provide a super massive BH formation mechanism allowing us to deal with the most massive central dark objects of $M_{BH} \sim 10^9 M_\odot$ found at the center of AGNs. This can be achieved by taking the free parameters β_0 and θ_0 of the model, as close as possible to the relativistic regime, at the point of the gravitational instability, as clearly shown in [25]. For this, by fixing the fermion mass m in the keV regime, the different solutions of the model, are systematically constructed along the one-parameter sequences of equilibrium configurations up to the critical point, which is represented by the maximum in a central density (ρ_0) Vs. core mass (M_c) diagram. At this stage, the compact cores reach the Oppenheimer-Volkoff (OV) mass limit given by the usual equation $M_c^{cr} \propto m_{pl}^3/m^2$, which is of the order of $M_c \sim 10^9 M_\odot$ for an ino mass $m \sim 10$ keV/ c^2 , with m_{pl} the planck mass. Clearly, this approach makes the use of a General Relativistic treatment to be mandatory.

⁸It is important to notice that our definition of halo radius r_h at the onset of the rotation curve (second maximum of it), implies $r_h > r_0$, and then, it is necessary to give precisely r_0 in each case to properly compare the theoretical model with (8). For this it was calculated the corresponding Burkert-like radius r_0 of each theoretical profile according to the Burkert prescription $\rho(r_0) = \rho_{0h}/4$. The very close behaviour between our cored halo density profile and the Burkert one (see Figure 2 and also [28]) is fully justifying this procedure.

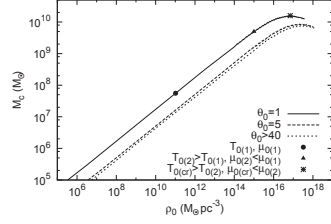


FIGURE 4. Different sequences of equilibrium configurations plotted in a (ρ_0) Vs. (M_c) diagram. The critical core mass is reached at the maximal value of M_c showing an appreciable increase up to $\sim 10^{10} M_\odot$ due to the temperature effects.

θ_0	β_0^{cr}	μ_0^{cr}/mc^2	$M_c^{cr}(M_\odot)$
1	6.45×10^{-2}	6.45×10^{-2}	1.59×10^{10}
5	2.23×10^{-2}	1.11×10^{-1}	7.91×10^9
40	8.33×10^{-3}	3.33×10^{-1}	7.44×10^9
55	6.06×10^{-3}	3.33×10^{-1}	7.44×10^9
100	3.33×10^{-3}	3.33×10^{-1}	7.44×10^9

TABLE 2. Critical temperature parameter and normalized chemical potential at the center of each different critical configuration, for different fixed central degeneracies. In the last column the critical core mass just before collapsing to a black hole.

We present here the sequence of equilibrium solutions obtained from Equation (4), for fixed particle mass $m \sim 10 \text{ keV}/c^2$, with initial conditions $M(0) = \nu(0) = 0$, given parameters $\theta_0 > 0$ (depending on the chosen central degeneracy), and temperature parameters β_0 until its critical value β_0^{cr} when the relativistic regime is reached. The different thermodynamic equilibrium configurations are thus plotted in a central density (ρ_0) vs. core mass (M_c) diagram (see Figure 4), where each point in the sequence has different central temperatures T_0 and central chemical potential μ_0 , so that satisfy the θ_0 fixed condition. The critical core mass $M_c^{cr} \sim 10^9 M_\odot$ is reached at the maximum of each $M_c(\rho_0)$ curve.

The critical points along each family of thermodynamic equilibrium configurations, fulfills the *turning point* definition given in [49], showing the onset of a thermodynamic instability on one side of the turning point. In Table 2 we show a set of central critical parameters of the model together with the correspondent critical core masses, for a very wide range of fixed central degeneracy parameters θ_0 , and fixed particle mass $m \sim 10 \text{ keV}/c^2$. It can be seen that for fully degenerate cores ($\theta_0 \gg 1$), the OV mass limit $M_c^{cr} \propto M_{pl}^3/m^2$ is numerically obtained, while instead for low degenerate cores, the critical core mass increases showing the temperature effects in a non linear way. See also [25] for details and further discussions on possible astrophysical implications of this model regarding a relation between super massive BHs and simultaneous co-existence of the DM halo depending on the cosmological epoch z .

CONCLUSIONS

We have presented a novel approach to calculate DM equilibrium configurations on galaxy scales based on the natural assumption of a late-time microscopic phase-space density of DM in a virialized halo $f(r, p)$ of Fermi-Dirac type, supported by collisionless relaxation formation mechanisms well studied in the last decades. The main result of this new picture is the prediction of a novel core-halo distribution of DM in galaxies based on fermionic quantum statistics and gravitational interactions. This model has been contrasted phenomenologically with galactic observables ranging from dwarf to big spiral galaxies, evidencing a fermion mass of about 10 keV as the more natural candidate, and implying: i) DM profiles in agreement with the observed constant and Universal DM halo surface density; ii) the arising of dense central quantum cores which may work as an alternative to intermediate ($M_c \sim 10^4 M_\odot$) and massive black holes (up to $M_c \sim 10^7 M_\odot$) in simultaneous compatibility with each observed DM halo; iii) a universal correlation between the halo mass M_h and the dark compact core mass M_c found from first principle physics and without assuming the central BH hypothesis, which is comparable with observations. We have further evaluated through a phenomenological approach, the role of a possible self-interacting nature for the dark matter particles in the Galaxy giving special attention to the effects in the dense and cold quantum cores. For the first time in the context of dark matter on galaxy scales, we have explicitly developed a theoretical framework within a (relativistic) field microscopic model to include the physics of self-interactions; new physics which has been already found in laboratory experiments in the context of ultra-cold atomic collisions in (effective) Fermi gases, motivating our original approach. Finally, it is also shown how the keV fermionic model can also be applied to provide a formation mechanism of SMBHs of $\sim 10^9 M_\odot$ as found in the center of active galaxies; providing in total, the first steps through a more unified picture that allow us to understand the deep connection between the particle DM and the small scale structures of the Universe.

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